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Quantisation in a non-linear gauge and Feynman rules

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Abstract. Certain aspects of the use of a non-linear gauge fixing condition in SU(N) gauge field theory are studied. The Faddeev-Popov ghost Lagrangian is obtained systematically using two different but related methods. Feynman rules are explicitly derived from the generating functional for the full Lagrangian in non-linear gauge. The tree-level amplitude for gauge field-gauge field scattering is shown to be independent of the parameter signalling the non-linearity of the gauge fixing condition and hence the same as the one in the linear covariant (Lorentz) gauge. The full Lagrangian is shown to have BRS invariance and the explicit form for BRS transformations is given. Using this, the corresponding Slavnov-Taylor identities are derived. The Lagrange multiplier formalism is used to derive the ghost Lagrangian independently and shown to be the same as that obtained by the gauge variation of the gauge fixing condition. The corresponding situation in Abelian gauge theories is reviewed. The coupling of the ghosts with gauge fields and Gribov ambiguity are discussed.

1. Introduction

With the advent of gauge theories and their remarkable success in describing the physics of elementary particles, it has become necessary to re-examine some of the established ideas in quantum field theories. It is well known that the quantisation of a gauge theory, either in the canonical formalism or in the path integral approach, involves the gauge fixing condition in a non-trivial way. In Abelian gauge theories, like the electromagnetic field, the essential role played by the gauge fixing condition is to eliminate the unphysical degrees of freedom of the photon field and there are [1] well defined prescriptions for carrying out this programme. In the path integral approach, the gauge fixing condition introduces fictitious ghost fields which do not couple with the gauge field. This statement, however, is not true if one employs a non-linear gauge fixing condition for the electromagnetic field, as we shall see later. Further, the gauge fixing condition in Abelian theories is unique in the sense that there are no Gribov ambiguities. Again, this statement is true only when one uses a linear gauge fixing condition at zero temperature. Use of non-linear gauges gives rise to Gribov ambiguity even at zero temperature. At non-zero temperature it has been shown that there is Gribov ambiguity if one employs a covariant gauge fixing condition [2] such as the Lorentz gauge, while such an ambiguity does not exist if one uses non-covariant gauges such as the Coulomb gauge. In quantising the non-Abelian theories, the gauge fixing condition introduces ghost fields which couple with the non-Abelian gauge fields in a non-trivial way. Further, whatever one chooses for the gauge fixing condition (except for algebraic gauges) Singer [3] proved that there will always be the Gribov ambiguity. Following the usual wisdom [4], one realises that the existence of Gribov ambiguities does not bother us as far as the perturbative sector is concerned.

Quite apart from the difficulties of the Gribov ambiguity in non-Abelian gauge theory, the question of finding the correct Lagrangian for the fictitious ghost fields has been raised and discussed in the mid-1970s, since the fundamental work of Feynman [5]. By the correct Lagrangian, we mean the one which eventually leads to unitarity of the S matrix. There are two different but related formalisms for the treatment of fictitious ghost fields. In the first method given by Faddeev and Popov [6] and discussed by DeWitt [7], Mandelstam [8], 't Hooft [9], Lee and Zinn-Justin [10] and Hsu [11] one uses the gauge fixing condition and its variation under general gauge transformations to obtain fictitious fields and their interaction with the gauge fields. In the second method, due to Hsu and Sudarshan [12], one employs the method of Lagrange multiplier fields and the equations of motion that follow to introduce the fictitious fields. In most cases studied the two methods give an identical Lagrangian for the fictitious fields which restore both unitarity and gauge invariance. The central role played by gauge theories in our understanding of elementary particle interactions provides the motivation for a detailed study of the quantum Yang–Mills theory in a variety of gauges. Christ [13] thus studied the operator ordering and Feynman rules in general non-covariant gauges.

It is interesting to note that most works in non-Abelian theories involve only linear gauge fixing conditions. It has been taken for granted [14] that there will be no difficulty in quantising a gauge theory in a non-linear gauge and that the resulting Feynman rules will be nearly the same. So it is worthwhile to examine systematically gauge theory in non-linear gauges. Hsu and Sudarshan [12] have considered a particular non-linear (more precisely bi-linear) gauge fixing condition for the spontaneously broken electroweak gauge theory in which some of the gauge bosons ($W^{\pm}$, Z) acquired mass and demonstrated the advantages of their Lagrange multiplier formalism, over the first method, in bringing out the physical reasons for the restoration of the unitarity of the S matrix. 't Hooft and Veltman [15] have employed a non-linear gauge fixing of the type

$$\mathscr{F} = \partial_\mu A^\mu + \lambda A_\mu A^\mu \tag{1}$$

for quantising the electromagnetic field (an Abelian theory). They obtain the Feynman rules for QED in this gauge which are quite *different* from the usual ones. In particular, the photon field couples with the ghost field and has cubic and quartic couplings with itself. However the photon–photon scattering amplitude at the tree level has been shown to be zero, as it should be, despite these exotic couplings. By using Ward identities the amplitude is shown to be independent of λ at one-loop level.

With this in mind, and with the motivation of studying non-Abelian gauge theory in a non-linear gauge, we consider in this paper a non-Abelian generalisation of the non-linear gauge fixing condition (1) by taking

$$\mathscr{F}^a = \partial_\mu A^{\mu a} + \lambda d^{abc} A^b_\mu A^{\mu c} \tag{2}$$

with d^{abc} as the symmetric coefficients of SU(N) algebra, $N > 2$. This is a natural generalisation of (1) and is different from the bi-linear gauge conditions examined in [12]. The motivation here is (a) to examine the non-linear gauges systematically for non-Abelian theory as has been done in [15] for the Abelian case, (b) to explicitly derive the Feynman rules in this case, (c) to check whether the gauge field–gauge field scattering amplitudes are independent of λ, (d) to examine whether or not the full

Lagrangian has BRS invariance and (*e*) to examine whether the fictitious ghost Lagrangians obtained by the aforementioned two methods are the same, and thereby unique. At this stage we have in mind no specific application of this to a process. Presumably, non-perturbative processes can be studied in this gauge. Therefore we consider a general SU(N) gauge theory.

The paper is organised as follows. In § 2 the use of the non-linear gauge (1) for Abelian theory is briefly reviewed. In § 3 non-Abelian gauge theory, based upon the gauge group SU(N), is studied using gauge (2) by employing the first method, namely the Faddeev-Popov formalism. The Feynman rules are derived and displayed. The tree-level on-mass shell amplitude for gauge field-gauge field scattering is shown to be independent of λ. In § 4 the full Lagrangian is shown to have BRS invariance and an expression for the conserved BRS current is derived. Using BRS invariance, Slavnov-Taylor identities are obtained which can be used to show the independence of the gauge field-gauge field scattering amplitude on λ by following the steps in [15]. In § 5 the Lagrange multiplier formalism is used to derive the ghost Lagrangian, and then is applied to the case under consideration and to Abelian theory. The resulting Lagrangian turns out to be identical to the one derived in § 3. Section 6 contains a summary.

2. Review of Abelian gauge theory in a non-linear gauge

Consider the Lagrangian density for U(1) gauge theory:

$$\mathscr{L}_0 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

which is invariant under the gauge transformation $A_\mu \to A_\mu + \partial_\mu \Lambda$. For the reasons given in the introduction, we fix the gauge

$$\mathscr{F}(A) = 0 \tag{4}$$

where $\mathscr{F}(A)$ is taken to be a non-linear differential equation for A_μ. Following the standard procedure [4, 14] the generating functional is given by

$$Z[0] = \int [\mathrm{d}A_\mu]\Delta[A_\mu] \exp\left[\mathrm{i}\int \mathrm{d}^4x\left(\mathscr{L}_0 - \frac{1}{2\alpha}\mathscr{F}^2\right)\right] \tag{5}$$

with the gauge invariant $\Delta^{-1}[A_\mu]$ as

$$\int [\mathrm{d}\Lambda]\delta\mathscr{F}[A_\mu]. \tag{6}$$

It can be shown [4, 14] that $\Delta[A_\mu] = \det M$, where M is the operator $\delta\mathscr{F}/\delta\Lambda$. It is essentially the gauge variation of the gauge fixing condition. Exponentiating $\det M$ by introducing anticommuting fields η, $\bar{\eta}$, one has

$$Z[0] = \int [\mathrm{d}A_\mu][\mathrm{d}\eta][\mathrm{d}\bar{\eta}] \exp\left(\mathrm{i}\int \mathrm{d}^4x(\mathscr{L}_0 + \mathscr{L}_{\mathrm{GF}} + \mathscr{L}_{\mathrm{FPG}})\right) \tag{7}$$

where $\mathscr{L}_{\mathrm{GF}} = -(1/2\alpha)\mathscr{F}^2$ and $\mathscr{L}_{\mathrm{FPG}}$ can be determined once $\mathscr{F}$ is known. We now adopt this as the usual procedure [6] for $\mathscr{F}$ given by (1). A calculation immediately gives

$$\delta\mathscr{F}/\delta\Lambda = \partial_\mu\partial^\mu + 2\lambda A_\mu\partial^\mu$$

and so

$$\mathscr{L}_{\mathrm{FPG}} = -\bar{\eta}\partial_\mu \partial^\mu \eta - 2\lambda \bar{\eta} A_\mu \partial^\mu \eta \tag{8}$$

which is the ghost Lagrangian for U(1) gauge theory in the non-linear gauge (1). This agrees with (11) and (12) of 't Hooft and Veltman [15] wherein (8) is shown to reflect the internal structure of the non-local Bell–Treiman transformation

$$A_\mu \to A_\mu - \mathrm{i}\lambda \partial_\mu \int \mathrm{d}^4 x' \, \Delta(x - x') A_\nu(x') A^\nu(x')$$

with

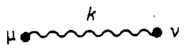
$$\Delta(x - x') = \frac{1}{(2\pi)^4 \mathrm{i}} \int \frac{\mathrm{d}^4 k}{k^2} \exp[\mathrm{i}k(x - x')] \tag{9}$$

used to go from the gauge $\mathscr{F} = \partial_\mu A^\mu$ to (1). Thus the full Lagrangian density is

$$\mathscr{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2}(\partial_\mu A^\mu + \lambda A_\mu A^\mu)^2 - \bar{\eta}\partial_\mu \partial^\mu \eta - 2\lambda \bar{\eta} A_\mu \partial^\mu \eta. \tag{10}$$

Contrary to the usual treatment of U(1) gauge theory with a linear gauge fixing condition, we have ghosts coupled with the photon field and the photon field has cubic and quartic couplings through the non-linear gauge fixing condition. The Feynman rules [15] from (10) are given below for the sake of completion and comparison with the similar rules in § 3.

(*a*) Photon propagator



$$\frac{1}{\mathrm{i}(2\pi)^4} \frac{g_{\mu\nu}}{k^2 - \mathrm{i}\varepsilon}.$$

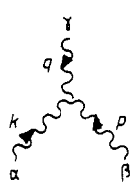
(*b*) Ghost propagator

$$\frac{1}{\mathrm{i}(2\pi)^4} \frac{1}{k^2 - \mathrm{i}\varepsilon}.$$

(*c*) Ghost–photon vertex

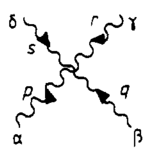
$$-(2\pi)^4 2\lambda k_\mu.$$

(*d*) Photon cubic vertex



$$2\lambda (2\pi)^4 (g_{\alpha\beta} q_\gamma + g_{\alpha\gamma} p_\beta + g_{\beta\gamma} k_\alpha) \delta(p + q + k).$$

(*e*) Photon quartic vertex



$$-4\lambda^2 \mathrm{i}(2\pi)^4 (g_{\alpha\beta} g_{\gamma\delta} + g_{\alpha\gamma} g_{\beta\delta} + g_{\alpha\delta} g_{\beta\gamma}) \delta(p + q + r + s).$$

With these Feynman rules it can be verified that the photon–photon scattering amplitude at tree level vanishes identically, with gauge-invariant external sources and on the mass shell. 't Hooft and Veltman [15] have obtained the Ward identities for the Lagrangian (9) which can be used to show the independence of the scattering amplitude on λ. The derivation of the Lagrangian for the ghosts (8) using the Hsu–Sudarshan formalism will be taken up in § 5. Here we wish to point out the following. Firstly, the gauge fixing condition (1) for an Abelian theory is not unique. Gauge transforming A_μ, the new gauge field obeys the same gauge fixing condition only if the gauge function Λ obeys

$$\Box\Lambda + 2A_\mu\partial^\mu\Lambda + (\partial_\mu\Lambda)(\partial^\mu\Lambda) = 0. \tag{11}$$

In a linear gauge, such as the Lorentz gauge, the corresponding equation will simply be $\Box\Lambda = 0$ which has the unique solution $\Lambda = 0$ if one demands Λ to be non-singular and vanishing as $x \to \infty$. In the present context the above Gribov equation may have a non-trivial solution for Λ. This would then mean that the gauge fixing condition is not unique. Thus the statement that Abelian gauge theories will not have a Gribov ambiguity is true only for linear gauges. Even working with linear gauges, it has been shown [2] that there will be a Gribov ambiguity in an Abelian gauge theory at finite temperature if one uses gauges that are defined on the full space ($\mathbb{R}^3 \times S^1$) such as the Lorentz gauge (covariant gauge). There, however, the use of the Coulomb gauge, which is defined on $\mathbb{R}^3$ alone, will not give rise to a Gribov ambiguity. Thus the existence of a Gribov ambiguity depends upon the gauge fixing condition as well as the gauge group. Secondly, the statement that in Abelian theories ghosts decouple from the gauge fields is not generally true. They do couple with the gauge fields in non-linear gauges. Similarly, photons can also have cubic and quartic couplings in non-linear gauges. Nevertheless, the relevant quantities, such as the S matrix elements, are independent of such unusual couplings.

3. SU(N) Gauge theory in a non-linear gauge ($N > 2$)

Consider the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{12}$$

where f^{abc} are the (antisymmetric) structure constants of the SU(N) group. The above Lagrangian is invariant under the gauge transformation

$$A^a_\mu \to A^a_\mu + \delta A^a_\mu$$
$$\delta A^a_\mu = D^{ab}_\mu \omega^b \qquad \omega^a \in \mathrm{SU}(N) \tag{13}$$
$$D^{ab}_\mu = \partial_\mu \delta^{ab} + g f^{acb} A^c_\mu .$$

The gauge fixing condition is chosen to be

$$\mathscr{F}^a = \partial_\mu A^{\mu a} + \lambda\, \mathrm{d}^{abc} A^b_\mu A^{\mu c}. \tag{2}$$

We first follow the Faddeev–Popov procedure to find $\mathscr{L}_{\text{ghost}}$. Accordingly, the gauge variation of (2) gives

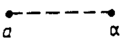
$$\delta\mathscr{F}^a/\delta\omega^\alpha = \Box\delta^{a\alpha} - g\lambda f^{ac\alpha} d^{cef} A^e_\mu A^{\mu f} + g f^{ac\alpha} A^c_\mu \partial^\mu + 2\lambda d^{a\alpha c} A^c_\mu \partial^\mu + 2\lambda g d^{abc} f^{be\alpha} A^e_\mu A^{\mu c}. \tag{14}$$

The determinant of this operator is the familiar Faddeev–Popov determinant Δ_{FP} which when exponentiated gives the Faddeev–Popov ghost Lagrangian as

$$\mathscr{L}_{\mathrm{FPG}} = -\bar{\eta}^{a}\partial^{\mu}D^{a\alpha}_{\mu}\eta^{\alpha} - \bar{\eta}^{a}2\lambda d^{abc}A^{c}_{\mu}D^{\mu b\alpha}\eta^{\alpha}. \tag{15}$$

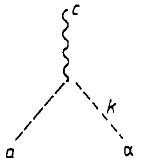
Before proceeding to the full Lagrangian to derive the Feynman rules, we note that to determine the ghost–gauge field coupling the $\partial^{\mu}A^{c}_{\mu}$ part in the first term of (15) is rewritten using (2). Then we obtain the following Feynman rules for the ghost sector.

(*a*) Ghost propagator:

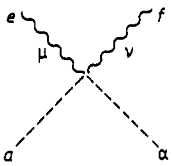


$$\frac{1}{\mathrm{i}(2\pi)^4}\frac{\delta^{a\alpha}}{k^2-\mathrm{i}\varepsilon}.$$

(*b*) Ghost–gauge field coupling:



$$-(2\pi)^4(gf^{ac\alpha}k_{\mu}+2\lambda d^{a\alpha c}k_{\mu}).$$



$$2\lambda g(2\pi)^4(d^{abf}f^{be\alpha}+d^{abe}f^{bf\alpha}+d^{bef}f^{ba\alpha})g_{\mu\nu}.$$

In the first diagram in (*b*) there is an additional contribution from the λ term in the gauge fixing condition. The second diagram (*b*) is not present in the usual case and appears to be a new diagram. Nevertheless, by using the Jacobi identity

$$[T_a,\{T_b,T_c\}]+\text{cyclic}=0$$

we find that $d^{abf}f^{be\alpha}+d^{abe}f^{bf\alpha}+d^{bef}f^{ba\alpha}=0$ and hence this diagram gives zero contribution.

The full Lagrangian is thus

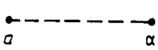
$$\mathscr{L} = -\tfrac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\alpha}(\partial_{\mu}A^{\mu a}+\lambda d^{abc}A^{b}_{\mu}A^{\mu c})^2 - \bar{\eta}^{d}\partial_{\mu}D^{\mu ab}\eta^{b} - 2\lambda\bar{\eta}^{a}d^{abc}A^{c}_{\mu}D^{\mu b\alpha}\eta^{\alpha} \tag{16}$$

which is the desired solution to the quantisation of SU(N) Yang–Mills theory in the non-linear gauge (2). We have not considered the matter fields here. They can be added by the gauge-invariant prescription. We now write down the complete Feynman rules.

(*a*) Propagator for gauge field:

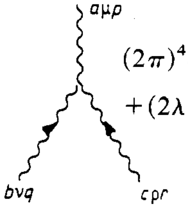
$$\frac{1}{\mathrm{i}(2\pi)^4}\frac{\delta^{ab}}{(k^2-\mathrm{i}\varepsilon)}\left(g_{\mu\nu}+(\alpha-1)\frac{k_{\mu}k_{\nu}}{k^2}\right).$$

(*b*) Ghost propagator:



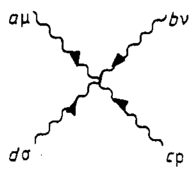
$$\frac{1}{\mathrm{i}(2\pi)^4}\frac{\delta^{a\alpha}}{(k^2-\mathrm{i}\varepsilon)}.$$

(*c*) Gauge field cubic coupling:



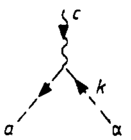
$$(2\pi)^4\delta(p+q+r)\{gf^{abc}[(r-q)_\mu g_{\nu\rho}+(p-r)_\nu g_{\mu\rho}+(q-p)_\rho g_{\mu\nu}]$$
$$+(2\lambda/\alpha)d^{abc}(p_\mu g_{\nu\rho}+q_\nu g_{\mu\rho}+r_\rho g_{\mu\nu})\}.$$

(*d*) Gauge field quartic coupling:



$$-(2\pi)^4 \mathrm{i}\delta(p+q+r+s)\{g^2[f^{ebc}f^{ead}$$
$$\times(g_{\mu\nu}g_{\sigma\rho}-g_{\mu\rho}g_{\sigma\nu})+f^{edb}f^{eac}(g_{\mu\sigma}g_{\rho\nu}-g_{\mu\nu}g_{\sigma\rho})$$
$$+f^{ecd}f^{eab}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})]+(4\lambda^2/\alpha)(d^{ead}d^{ebc}g_{\sigma\mu}g_{\rho\nu}$$
$$+d^{eac}d^{ebd}g_{\sigma\nu}g_{\rho\mu}+d^{eab}d^{edc}g_{\mu\nu}g_{\sigma\rho})\}.$$

(*e*) Ghost-gauge field coupling:



$$-(2\pi)^4[gf^{ac\alpha}k_\mu+2\lambda d^{ac\alpha}k_\mu].$$

These are the complete Feynman rules in the non-linear gauge (2). (The notation and convention for diagrams are the same as in [15].) This is one of the main results of this paper. Fermions can be added in the conventional manner whose Feynman rules have been given in the literature [4].

Let us now consider the gauge field-gauge field scattering amplitude at tree level. In the case of Abelian gauge theory, the photon-photon scattering amplitude is zero at tree level in linear gauges. In § 2 and in [15] such amplitudes remain zero in the non-linear gauge despite the aforementioned self-couplings of the photons. However, in the case of non-Abelian gauge theory, the corresponding amplitude does not vanish at tree level in linear gauges owing to the inherent nature of the self-couplings of the Yang–Mills fields. Employing the non-linear gauge (2), these self-couplings involve the parameter λ non-trivially, as can be seen from the Feynman rules. Here we would like to examine whether this amplitude is independent of λ under the same gauge-invariant couplings with the external sources used in [15]. At tree level there are four diagrams (*a*), (*b*), (*c*) and (*d*) that contribute to the scattering amplitude. These, for the Abelian case, are given in [15, p 61]. We will not reproduce the diagrams here, but give the results only. To fix the notation we label the legs of diagram (*a*) of [15, p 61] as $(a\mu p)$, $(b\nu q)$, $(c\rho r)$ and $(d\sigma s)$ clockwise starting from the left-hand top leg, where a, b, c, d are group indices, μ, ν, ρ, σ are Lorentz indices and p, q, r, s are the respective momenta. The diagrams (*b*), (*c*) and (*d*) of the above [15] are similarly labelled. Then the amplitudes for (*a*), (*b*) and (*c*) involve g^2, λ^2 and λg terms while that for (*d*) involves only g^2 and λ^2 terms. Diagram (*d*) is simply the quartic coupling diagram given above. The λ^2 terms from (*a*), (*b*) and (*c*), respectively, are (choosing $\alpha=1$) $\mathrm{i}(2\pi)^4 4\lambda^2 d^{ade}d^{bec}g_{\mu\sigma}g_{\nu\rho}$, $\mathrm{i}(2\pi)^4 4\lambda^2 d^{edc}d^{eba}g_{\sigma\rho}g_{\nu\mu}$ and $\mathrm{i}(2\pi)^4 4\lambda^2 d^{edb}d^{eac}g_{\sigma\nu}g_{\mu\rho}$ which, when added, cancel the λ^2 term for diagram (*d*). The λg terms for each of the (*a*), (*b*) and (*c*) diagrams vanish due to the massless nature of the gauge fields. For example, the term for diagram (a) has the form

$$-d^{ead}f^{bec}g_{\mu\sigma}g_{\rho\nu}+d^{ead}f^{bec}k_{\lambda}g_{\mu\sigma}(-2r_{\nu}g_{\lambda\rho}+2q_{\lambda}g_{\rho\nu}-2q_{\rho}g_{\nu\lambda})/k^2$$
$$+d^{bec}f^{ade}g_{\mu\sigma}g_{\nu\rho}+d^{bec}f^{ade}k_{\lambda}g_{\nu\rho}(-2s_{\mu}g_{\sigma\lambda}-2p_{\sigma}g_{\mu\lambda}+2p_{\lambda}g_{\mu\sigma})/k^2 \qquad (17)$$

where k is the 4-momentum of the exchanged gauge field and we have $p+k=s$ and $q=k+r$, the momentum conservation at each vertex. Using these momentum conservations and the massless nature of the gauge fields ($q^2=p^2=r^2=s^2=0$) this amplitude vanishes identically. Similarly, the λg terms from diagrams (b) and (c) separately vanish. Thus the gauge field-gauge field scattering amplitude at tree level is shown to be independent of λ. Unlike the Abelian case, the tree-level amplitude does not vanish due to g^2 terms. The explicit expression for this is the same as in the Lorentz gauge and so we do not give this here. The main point here is that the use of a non-linear gauge gives the same expression for the above scattering amplitude as if we were working in a linear gauge, such as the Lorentz gauge. However, linear gauges are simple to use. Non-linear gauges can also be employed and, except for their lack of simplicity, non-linear gauges are equally as good as the linear gauges.

We close this section with two observations relevant to non-linear gauges. First, it has already been pointed out that in Abelian gauge theory there will be Gribov ambiguity when non-linear gauges (1) are employed. In the case of non-Abelian gauge theory there will always be Gribov ambiguity, even if linear gauges are employed [3]. In non-linear gauges (2) for non-Abelian theories, there is surely Gribov ambiguity, thereby indicating that the gauge fixing condition is not unique, reflecting the general behaviour of the non-Abelian theories. For (2), requiring the gauge transformed A_{μ}^{a} to obey the non-linear gauge fixing condition, we have the following differential equation:

$$\partial_{\mu}(D^{\mu ab}\omega^{b})+2\lambda d^{abc}A_{\mu}^{b}(D^{\mu cd}\omega^{d})+\lambda d^{abc}(D_{\mu}^{bd}\omega^{d})(D^{\mu ce}\omega^{e})=0 \qquad (18)$$

for the gauge function ω^a. This is the Gribov equation in our case and should be compared with (11) for Abelian gauge theory. With $\lambda=0$ the usual Gribov equation is obtained. The above equation is non-linear in ω^a and may have interesting solutions whose existence reveals the non-uniqueness of the gauge fixing condition. Thus the non-linear gauge shares this non-uniqueness property with linear gauges. Second, one can generalise the Bell–Treiman transformation (9) to non-Abelian theories. The generalised form is

$$A_{\mu}^{a}\rightarrow A_{\mu}^{a}+\lambda d^{abc}\partial^{-2}\partial_{\mu}(A_{\nu}^{b}A^{\nu c}) \qquad (19)$$

so that

$$\partial_{\mu}A^{\mu a}\rightarrow\partial_{\mu}A^{\mu a}+\lambda d^{abc}A_{\nu}^{b}A^{\nu c}.$$

Thereby one can go from the Lorentz (linear) gauge to the non-linear gauge (2). Explicitly:

$$A_{\mu}^{a}\rightarrow A_{\mu}^{a}-\mathrm{i}\lambda d^{abc}\partial_{\mu}\int \mathrm{d}^4x'\,\Delta(x-x')A_{\nu}^{b}A^{\nu c} \qquad (20)$$

with $\Delta(x-x')$ given by (9). One can effectively use this generalised Bell–Treiman transformation to derive the ghost Lagrangian following the procedure of 't Hooft and Veltman [15]. In this paper we have preferred to use the standard Faddeev–Popov procedure. In § 5 we give another procedure using Lagrange multiplier fields.

4. BRS invariance and a non-linear gauge

From the results of § 3 we have the full Lagrangian as

$$\mathscr{L}_{\text{full}} = \mathscr{L}_0 + \mathscr{L}_{\text{GF}} + \mathscr{L}_{\text{FPG}} \tag{21}$$

with

$$\begin{aligned}
&\mathscr{L}_0 = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}\\
&\mathscr{L}_{\text{GF}} = -(1/2\alpha)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})^2\\
&\mathscr{L}_{\text{FPG}} = -\bar{\eta}^a(\delta^{ae}\partial_\mu + 2\lambda d^{ace}A^c_\mu)D^{\mu eb}\eta^b.
\end{aligned} \tag{22}$$

The generating functional is

$$Z[0] = \int [\mathrm{d}A_\mu][\mathrm{d}\eta][\mathrm{d}\bar{\eta}]\exp\left(\mathrm{i}\int \mathrm{d}^4x\mathscr{L}_{\text{full}}\right). \tag{23}$$

The local Lorentz covariant $\mathscr{L}_{\text{full}}$ gave the necessary Feynman rules displayed in § 3. One can instead use the measure $[\mathrm{d}A_\mu]\ \Delta_{\text{FP}}[A]$ to define the generating functional. The functional in (23) corresponds to a definite choice of the gauge fixing condition (2) and so the effective action $\int \mathrm{d}^4x\mathscr{L}_{\text{full}}$ is not gauge invariant. However Becchi *et al* [16] have shown that there exist transformations affecting simultaneously both the non-Abelian gauge fields and the fictitious ghost fields η, $\bar{\eta}$ which leave the full Lagrangian invariant. These transformations for linear gauge fixing conditions (which introduce specific $\mathscr{L}_{\text{FPG}}$) have been given in [4, 14]. One of the purposes of these super transformations which mix commuting and anticommuting fields is to obtain a condition on Z so that Z remains gauge invariant—the Slavnov-Taylor identities. Here we demonstrate the existence of such BRS transformations for $\mathscr{L}_{\text{full}}$ in (21) and (22) which corresponds to the choice of non-linear gauge fixing (2). These BRS transformations are given by

$$\begin{aligned}
&\delta A^a_\mu = -(1/g)(D^{ab}_\mu\eta^b)\xi\\
&\delta\eta^a = -\tfrac{1}{2}f^{abc}\eta^b\eta^c\xi\\
&\delta\bar{\eta}^a = -(1/\alpha g)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})\xi
\end{aligned} \tag{24}$$

where ξ is a constant Grassmann parameter, anticommuting with the ghosts η and $\bar{\eta}$ and commuting with the gauge field A^a_μ. The full Lagrangian (21) and (22) will be seen to remain invariant under the transformations (24). Briefly $\mathscr{L}_0$ remains obviously invariant since $\delta A^a_\mu = -(1/g)(D^{ab}_\mu\eta^b)\xi$ is a subclass of the general gauge transformation $\delta A^a_\mu = D^{ab}_\mu\omega^b$ with $\omega^b \in \mathrm{G}$. The BRS variation of $\mathscr{L}_{\text{GF}}$ is given by

$$\begin{aligned}
\delta\mathscr{L}_{\text{GF}} = {}& (1/\alpha g)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})\partial_\mu(D^{\mu ab'}\eta^{b'})\xi\\
&+ (2\lambda/\alpha g)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})d^{ab'c'}(D^{b'e}_\mu\eta^e)A^{\mu c'}\xi
\end{aligned} \tag{25}$$

while that of the $\mathscr{L}_{\text{FPG}}$ is

$$\begin{aligned}
\delta\mathscr{L}_{\text{FPG}} = {}& -(1/\alpha g)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})\partial_\mu(D^{\mu ab'}\eta^{b'})\xi - (\delta\bar{\eta}^a)2\lambda d^{abc}A^c_\mu(D^{\mu bb'}\eta^{b'})\\
&- 2\lambda\bar{\eta}^a d^{abc}(\delta A^c_\mu)D^{\mu bb'}\eta^{b'}
\end{aligned} \tag{26}$$

using $\delta(D^{bb'}_\mu\eta^{b'}) = 0$ which can be easily verified using (24). The first term in $\delta\mathscr{L}_{\text{GF}}$ cancels the first term in $\delta\mathscr{L}_{\text{FPG}}$. In the second term in $\delta\mathscr{L}_{\text{GF}}$, ξ can be brought to the left of $D^{b'e}_\mu\eta^e$ with a sign change. Then, noting $\delta\bar{\eta}^a = -(1/\alpha g)(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c})\xi$, this second term in $\delta\mathscr{L}_{\rm GF}$ cancels the second term in $\delta\mathscr{L}_{\rm FPG}$. Thus the effective BRS change in the full Lagrangian is

$$\delta\mathscr{L}_{\rm full} = -2\lambda\bar{\eta}^a d^{abc}(\delta A_\mu^c)D^{\mu bb'}\eta^{b'}. \tag{27}$$

Upon substituting for δA^c from (24), this becomes

$$-(2\lambda/g)\bar{\eta}^a\, d^{abc}(D_\mu^{cd}\eta^d)(D^{\mu be}\eta^e)\xi. \tag{28}$$

Since η^a is a Grassmann (anticommuting) field, $D_\mu^{ab}\eta^b$ will also be Grassmannian. Denoting $D_\mu^{ab}\eta^b$ by χ^a, we have $\delta\mathscr{L}_{\rm full} = -(2\lambda/g)\bar{\eta}^a\, d^{abc}\,\chi_\mu^c\chi^{\mu b}\xi$. As χ^b anticommutes with χ^c and d^{abc} is symmetric in b and c, the sum over b and c makes $\delta\mathscr{L}_{\rm full}$ vanish. (When $b=c$, $(\chi^b)^2=0$ for every b.) This proves the invariance of the full Lagrangian under the BRS transformation given in (24) for the non-linear gauge (2). It can be easily seen that the BRS transformations (24) are nilpotent.

Before proceeding to the derivation of Slavnov–Taylor identities, we now give the conserved BRS current so that the description may be complete. Following Nishijima [17] we define the current

$$J_\sigma = \sum_{a,\,i}\frac{\partial\mathscr{L}_{\rm full}}{\partial(\partial_\sigma\Phi_\rho^{ai})}\,\delta\Phi_\rho^{ai} \tag{29}$$

where Φ is any generic field in $\mathscr{L}_{\rm full}$, a is the SU(N) group index, ρ is the Lorentz index whenever necessary, the index i is for various fields present in $\mathscr{L}_{\rm full}$ and the summation is over all fields and the group index a. Straightforward evaluation then gives the conserved BRS current:

$$J_\sigma = (1/g)[-\partial_\rho(\eta^\alpha F_{\sigma\rho}^\alpha) - \eta^\alpha D^{\rho\alpha\beta}F_{\rho\sigma}^\beta] - (2/\alpha g)(D_\sigma^{\alpha\beta}\eta^\beta)(\partial_\mu A^{\mu\alpha} + \lambda d^{\alpha bc}A_\mu^b A^{\mu c})$$
$$-\tfrac{1}{2}f^{\alpha\beta\gamma}\eta^\beta\eta^\gamma(-\partial_\sigma\bar{\eta}^\alpha + 2\lambda d^{\alpha\alpha c}A_\sigma^c\bar{\eta}^a). \tag{30}$$

From this the BRS charge can be obtained as the spatial integral over J_0 which can be used to obtain the transformations (24).

We now take the generating functional (23) for consideration. It has been shown that the full Lagrangian is BRS invariant. We will introduce the sources for the fields A_μ^a, η^a, $\bar{\eta}^a$ and make a BRS transformation on the fields towards deriving the Slavnov–Taylor identities. First we show that the Jacobian of the transformation is unity. Writing

$$J_{ij} = \frac{\partial(A_\mu^a(x)+\delta A_\mu^a(x),\ \eta^a(x)+\delta\eta^a(x),\ \bar{\eta}^a+\delta\bar{\eta}^a(x))}{\partial(A_\nu^b(y),\ \eta^b(y),\ \bar{\eta}^b(y))}$$

so that

$$J_{11} = \frac{\partial(A_\mu^a(x)+\delta A_\mu^a(x))}{\partial A_\nu^b(y)} = (\delta^{ab}\delta_{\mu\nu} - f^{abc}\eta^c\xi\delta_{\mu\nu})\delta(x-y)$$

and similarly for $J_{12}, J_{13}, J_{21}, J_{22}, J_{23}, J_{31}, J_{32}$ and J_{33}, we find that $\det J = 1$. Introducing the sources, we have

$$Z[J,s,\bar{s}] = \int [{\rm d}A_\mu][{\rm d}\eta][{\rm d}\bar{\eta}]$$
$$\times\exp\left({\rm i}\int {\rm d}^4x(\mathscr{L}_0 + \mathscr{L}_{\rm GF} + \mathscr{L}_{\rm FPG} + J_\mu^a A^{\mu a} + \bar{\eta}^a s^a + \bar{s}^a\eta^a)\right). \tag{31}$$

The BRS transformations (24) are made on A_μ^a, η^a and $\bar{\eta}^a$. The measure does not change, and neither does $\mathscr{L}_0 + \mathscr{L}_{\rm GF} + \mathscr{L}_{\rm FPG}$. The source terms, however, do change.

After making the BRS transformation, following Faddeev and Slavnov [14], we equate the derivative $\mathrm{d}Z/\mathrm{d}\xi$ to zero, from which we obtain the following identity, after differentiating with respect to η and setting $\eta = \bar{\eta} = 0$:

$$\frac{1}{\alpha}\left[\partial_\mu\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta J^a_\mu}\right) + \lambda d^{abc}\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta J^b_\mu}\right)\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta J^c_\mu}\right)\right]Z(J)$$

$$= \int \mathrm{d}^4 Y J^{a'}_\mu D^{\mu a' b}\left(\frac{1}{\mathrm{i}}\frac{\delta}{\delta J_\mu}\right)G_{ba}(x, y) \tag{32}$$

where

$$M^{-1}_{ca} Z[J] = G_{ca}(x, y) \tag{33}$$

with $G_{ca}(x, y)$ as the ghost propagator in the presence of J and M_{ca} the differential operator given in (14) with A^a_μ replaced by

$$\frac{1}{\mathrm{i}}\frac{\delta}{\delta J^a_\mu}.$$

We close this section by stating that these identities can be used to show the independence of the gauge field-gauge field scattering amplitude on λ following 't Hooft and Veltman [15] wherein the Abelian case has been discussed. As the necessary steps are already given in [15] we do not repeat this straightforward calculation here.

5. Lagrange multiplier formalism

In the introduction we stated that there are two different but related formalisms for obtaining the Lagrangian for the ghosts in a gauge field theory. In § 3 we adopted the Faddeev-Popov method, based on the gauge variation of the gauge fixing condition, to obtain the $\mathscr{L}_{\mathrm{FPG}}$ for the non-linear gauge fixing condition (2). It would be worthwhile to examine whether the Faddeev-Popov formalism gives the correct $\mathscr{L}_{\mathrm{FPG}}$, in the sense that the theory so obtained is gauge invariant and unitary. Formal proofs to this effect have been given in [7-10]. Nonetheless, it is desirable, especially in view of the non-linear gauge (2), to obtain $\mathscr{L}_{\mathrm{FPG}}$ in a different formalism. Hsu and Sudarshan [12] proposed a different formalism based on the Lagrange multiplier fields.

We briefly review the Lagrange multiplier formalism now. The method consists of taking into account the gauge fixing condition through a Lagrange multiplier field. The field equations for the gauge field and the Lagrange multiplier field are obtained. These field equations are rearranged so that the multiplier field has a simple coupling with the gauge field. The gauge fields appearing in the gauge fixing condition are allowed to have a gauge transformation. This gives an equation for the gauge function. Detailed considerations [12] lead one to consider the multiplier field and the gauge function to correspond to the unphysical degrees of freedom of the gauge field. The two equations for the multiplier field and the gauge function completely determine the coupling of the unphysical components of the gauge field, and could be derived from a Lagrangian. In the generating functional, an integration over these two fields gives a factor which essentially gives the unitarisation factor, which is Δ_{FP}. This factor may be viewed as being generated by a fictitious ghost field. In contrast to the Faddeev-Popov method, this method uses extensively the field equations for the gauge field. The details concerning the unitarisation of the S matrix by this method are given in [11] and here we are mainly interested in getting the form of the Lagrangian for the ghosts.

For our purpose, we introduce a Lagrange multiplier field χ^a and consider the Lagrangian

$$\mathscr{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + (1/2\alpha)\chi^a(\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c}) + (1/8\alpha^2)\chi^a\chi^a. \tag{34}$$

This Lagrangian leads to the field equations

$$\begin{aligned} &\partial^\sigma F^a_{\sigma\rho} - (1/2\alpha)\partial_\rho\chi^a = gf^{abc}F^b_{\mu\rho}A^{\mu c} - (\lambda/\alpha)d^{abc}\chi^b A^c_\rho \\ &\partial_\mu A^{\mu a} + \lambda d^{abc}A^b_\mu A^{\mu c} + (1/2\alpha)\chi^a = 0. \end{aligned} \tag{35}$$

Using the second equation in the first, we get

$$\Box A^a_\mu + J^a_\mu = 0 \tag{36}$$

where

$$J^a_\mu = \lambda d^{abc}\partial_\mu(A^b_\nu A^{\nu c}) + gf^{abc}\partial_\nu(A^{\nu b}A^c_\mu) + gf^{abc}A^{\nu b}F^c_{\nu\mu} + (\lambda/\alpha)d^{abc}\chi^b A^c_\mu. \tag{37}$$

The divergence of (36) together with the second equation in (35) gives

$$(1/2\alpha)\Box\chi^a + \lambda d^{abc}\Box(A^b_\mu A^{\mu c}) = \partial_\mu J^{\mu a}. \tag{38}$$

Using (37), $\partial_\mu J^{\mu a}$ can be evaluated and then (38) becomes

$$\Box\chi^a = -gf^{abc}A^b_\mu(\partial^\mu\chi^c) + 2\lambda d^{abc}\partial_\mu(A^{\mu b}\chi^c) + 2\lambda g f^{abc}d^{cde}A^b_\mu A^{\mu e}\chi^d \tag{39}$$

which gives the coupling of the (unphysical) multiplier field with the gauge field. Following Hsu and Sudarshan [12], the source terms in the field equations for the Lagrange multiplier fields χ^a produce extra unwanted amplitudes that upset the unitarity. This is the dynamical origin of the extra amplitudes. The method used to cancel the extra amplitude is to introduce complex fictitious ghosts with the same couplings as given in (39). The fictitious ghost Lagrangian is constructed according to (39) and the gauge fixing condition (2). The other unphysical component (other than χ^a) is to be identified with the gauge function χ'^a which is given by

$$A^a_\mu \to A^a_\mu + D^{ab}_\mu\chi'^b.$$

The gauge function χ'^a must obey an equation so that the gauge transformed A^a_μ obeys again the same gauge fixing condition (2). This immediately leads to

$$\Box\chi'^a = -gf^{abc}\partial_\mu(A^{\mu b}\chi'^c) - 2\lambda d^{abc}A^b_\mu\partial^\mu\chi'^c - 2\lambda g d^{abc}f^{cde}A^b_\mu A^{\mu d}\chi'^e. \tag{40}$$

It will be realised that (39) coincides with the field equation for $\bar\eta^a$ and (40) with that for η^a obtained from $\mathscr{L}_{\text{FPG}}$ given by (15). A Lagrangian for the unphysical fields χ^a and χ'^a can be written as

$$\begin{aligned} \mathscr{L}(\chi,\chi') = {} & (\partial_\mu\chi^a)(\partial^\mu\chi'^a) + gf^{acb}(\partial_\mu\chi^a)A^{\mu c}\chi'^b \\ & - 2\lambda d^{abc}\chi^a A^c_\mu(\partial^\mu\chi'^b + gf^{bde}A^{\mu d}\chi'^e). \end{aligned} \tag{41}$$

To avoid the extra amplitude this must be added to the $\mathscr{L}_0$ and an integration over χ^a, χ'^a must be performed. This can be done easily to produce the unitarisation factor, the Faddeev–Popov determinant. This is exponentiated by introducing the anticommuting scalar fields which are precisely the same as those in (15). Thus we have shown that the method of Hsu–Sudarshan [12] based upon the Lagrange multiplier fields gives the same $\mathscr{L}_{\text{FPG}}$ (15) as obtained by the Faddeev–Popov method. Quite apart from the dynamical origin of Δ_{FP} in the second formalism, and the explicit method to isolate the unwanted amplitude, we would like to point out that this method explicitly makes use of the field equations for the A^a_μ field and in this sense consults $\mathscr{L}_0$ at every stage. This is to be contrasted with the Faddeev-Popov method which requires just the gauge variation of the gauge fixing condition to write down the ghost Lagrangian.

For the sake of completeness we now give the construction procedure for $\mathscr{L}_{FPG}$ for Abelian theory in the non-linear gauge (1). The gauge variation of the gauge fixing condition (1) gave

$$\mathscr{L}_{FPG} = -\bar{\eta}\partial_\mu\partial^\mu\eta - 2\lambda\bar{\eta}A_\mu\partial^\mu\eta \tag{8}$$

which incidentally can be obtained from (15) by setting $g=0$ and $d^{abc}=1$, removing the SU(N) indices. To use the Lagrange multiplier formalism of Hsu-Sudarshan [12] we introduce the multiplier field χ as

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\chi(\partial_\mu A^\mu + \lambda A_\mu A^\mu) + \tfrac{1}{8}\chi^2 \tag{42}$$

which leads to the field equations

$$\begin{aligned} &\partial^\mu F_{\mu\nu} = \tfrac{1}{2}\partial_\nu\chi - \lambda\chi A_\nu \\ &\partial_\mu A^\mu + \lambda A_\mu A^\mu + \tfrac{1}{2}\chi = 0. \end{aligned} \tag{43}$$

Differentiating the second equation with respect to x^ν and using in the first, we have

$$\begin{aligned} &\Box A_\mu + J_\mu = 0 \\ &J_\mu = 2\lambda A_\nu\partial_\mu A^\nu - 2\lambda A_\mu(\partial_\nu A^\nu + \lambda A_\nu A^\nu). \end{aligned} \tag{44}$$

Taking the divergence of the first equation in (44) and using (43), we obtain

$$\Box\chi = 2\lambda\partial_\mu(A^\mu\chi) \tag{45}$$

which gives the source for χ, coupling with A_μ which is the source for the violation of unitarity. Making the gauge transformation $A_\mu \to A_\mu + \partial_\mu\chi'$ on the non-linear gauge fixing condition (1), we have

$$\Box\chi' + 2\lambda A_\mu(\partial^\mu\chi') = 0. \tag{46}$$

Equations (45) and (46) completely determine the coupling of the unphysical components in $A_\mu(x)$. (For linear gauges, χ and χ' obey sourceless free-field equations and hence the statement that the ghosts eventually decouple from the gauge field.) These equations can be obtained from a Lagrangian

$$\mathscr{L}(\chi, \chi') = \chi\partial_\mu\partial^\mu\chi' + 2\lambda\chi A_\mu\partial^\mu\chi \tag{47}$$

which has the same form as (8). However, χ and χ' are not to be identified with ghosts. This Lagrangian has to be added to $\mathscr{L}_0$ and an integration over χ and χ' in the generating functional produces a determinant which will be the same as Δ_{FP}. Exponentiating this by introducing the anticommuting scalar fields will lead to (8).

It has thus been demonstrated that the ghost Lagrangian derived using the Faddeev-Popov procedure and the Hsu-Sudarshan procedure are the same for non-linear gauges in Abelian and non-Abelian theories. We hasten to add that we have not given the complete details of both the methods explaining how the unitarity is cured and how the physical states are to be constructed. Our purpose here is to bring out the form for the ghost Lagrangian in both the methods. Our demonstration at least indicates that the Faddeev-Popov procedure can be used for non-linear gauges as well.

6. Summary

Certain aspects of the non-linear gauge fixing condition in gauge field theories were given. Inspired by the use of the non-linear gauge (1) for Abelian theory, a generalisation of (1) for the non-Abelian theory (2) was considered. After reviewing the Abelian theory, the ghost Lagrangian for SU(N) gauge theory was constructed. Feynman rules were systematically derived from the full Lagrangian. This is one of our main results. The gauge field–gauge field scattering amplitude was shown to be independent of the parameter λ explicitly at tree level. So the amplitude was the same as if one was using the (linear) Lorentz gauge. A similar demonstration for an Abelian theory has been done by 't Hooft and Veltman [15]. The full SU(N) Lagrangian including $\mathscr{L}_{\text{GF}}$ and $\mathscr{L}_{\text{FPG}}$ was shown to have BRS invariance and the form of the BRS transformations and current were displayed. After showing the Jacobian for BRS change, for the measure in the generating functional being unity, the Slavnov–Taylor identities were derived, which can be used to consider one-loop amplitudes to demonstrate their gauge invariance. This completed the major part of the paper. Although the Faddeev–Popov procedure has been formally proved to give a unitary and gauge-invariant S matrix, we investigated the crucial $\mathscr{L}_{\text{FPG}}$ by appealing to a different procedure, developed by Hsu and Sudarshan. It is worth noting that this method effectively makes use of the field equations for the gauge field and Lagrange multiplier field and gives physical insight to the Faddeev–Popov determinant. By using this method, we demonstrated that the ghost Lagrangian obtained this way coincides with that obtained by the former method. The role of the Bell–Treiman transformation was briefly discussed. It was noted explicitly that the statement, that in Abelian theories the ghosts *decouple* from the gauge fields, is true *only* in *linear gauges*. In a non-linear gauge, they have a non-trivial coupling with the gauge fields. This can be clearly seen in (45) and (46), which give sources for the Lagrange multiplier fields. In a similar way, the statement that the Abelian gauge theories do not have Gribov ambiguity, or equivalently that one can have a unique gauge fixing condition, is gauge dependent. It means that in Abelian theories one can choose a unique linear gauge in which there are no Gribov ambiguities. The use of non-linear gauge in Abelian theories makes the non-uniqueness transparent. In non-Abelian theories, the non-linear gauge brings additional non-linear terms in the Gribov equation. For perturbative calculations, these ambiguities can be ignored. The Feynman rules derived here can be used for perturbative calculations and one such application shows that the results are identical with those obtained with Feynman rules for a linear gauge fixing condition.

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